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STELLAR REACTION RATES FOR ^{28}Si*

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ABSTRACT

Stellar rates for the reactions ^{27}Al$(p,\gamma)^{28}$Si and ^{24}Mg$(\alpha,\gamma)^{28}$Si have been calculated from new cross-section data for these reactions. The contributions to the interaction rates, $\langle\sigma v\rangle_{p\gamma}$ and $\langle\sigma v\rangle_{\alpha\gamma}$, and to the corresponding photodisintegration rates, $\lambda_{\gamma p}$ and $\lambda_{\gamma\alpha}$, from the excited states of ^{27}Al, ^{24}Mg, and ^{28}Si are discussed. The calculated interaction rates, $N_A\langle\sigma v\rangle$, are listed for temperatures from 0.1×10^{9} ° to 5.0×10^{9} ° K; two- and four-parameter fits are given for $N_A\langle\sigma v\rangle$ as a function of temperature.

I. INTRODUCTION

Recent investigations by Truran, Cameron, and Gilbert (1966) and by Bodansky, Clayton, and Fowler (1968) have focused interest on the silicon-burning stage of stellar evolution. These analyses have treated the evolution of a pure-silicon core by successive addition of α-particles and nucleons to the silicon. These α-particles and nucleons arise mainly from the photodisintegration of ^{28}Si and lighter α-particle nuclei at the high temperatures involved, $T_9 \geq 3$ (where T_9 is the temperature measured in units of 10^9 ° K). At these temperatures, a significant number of the ^{28}Si nuclei will be in excited states; thus it is important to know the *total* photodisintegration rate for all ^{28}Si nuclei, including those in excited states. Fowler and Hoyle (1964) have shown that such rates, including the effects of photodisintegration from excited states, can be determined from a knowledge of the *total* gamma resonance strengths for the inverse (particle-gamma) reaction.

A program was begun at this laboratory several years ago to examine several of the radiative-capture reactions which are the most critical in determining the time scale for the silicon-burning stage. While some of these reactions have been investigated by other experimenters, the earlier data are not always suitable for calculations of photodisintegration rates, either because the measured parameters were not those required for the calculations or because of inconsistencies among the measurements of various experimental groups. The detection system used in the present work consists of two large NaI crystals very close to a small target chamber; this system has a solid angle of nearly 4π and a high detection efficiency. An earlier paper (Lyons, Toevs, and Sargood 1969, hereinafter referred to as Paper I) describes the experimental apparatus and discusses the methods of data analysis. Paper I also describes the application of these techniques to a study of the reaction ^{27}Al$(p,\gamma)^{28}$Si, in the proton-energy range 0.3–2.6 MeV, and lists the resonance strengths for the ^{27}Al$(p,\gamma)^{28}$Si reaction. A second paper (Lyons 1969*a*, hereinafter referred to as Paper II) treats the ^{24}Mg$(\alpha,\gamma)^{28}$Si reaction in a similar manner, for α-particles of energy below 2.8 MeV.

Using the data given in Papers I and II, we present in this paper calculations of the photodisintegration rates, and empirical fits to these rates, for ^{28}Si in the environment postulated for the silicon-burning phase.

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II. THERMONUCLEAR REACTION RATES

a) *Interaction and Photodisintegration Rates*

Interaction rates $\langle\sigma v\rangle^0$, i.e., the mean product of the (p_0,γ) or (α_0,γ) cross-sections and velocity, have been calculated for the particle-capture reactions by using equations (32) and (62) of the review article by Fowler, Caughlan, and Zimmerman (1967, henceforth referred to as FCZ). The symbols p_0 and α_0 denote interactions of protons and α-particles with the ground states of the target nuclei ^{27}Al and ^{24}Mg, for the two reactions under discussion. For the (p_0,γ) case,

$$\langle\sigma v\rangle^0 = \left(\frac{2\pi\hbar^2}{MkT}\right)^{3/2}\Sigma_r \frac{g_{\mathrm{Si}}{}^r}{g_p g_{\mathrm{Al}}{}^0}\left(\frac{\Gamma_{p_0}\Gamma_\gamma}{\hbar\Gamma}\right)_r \exp(-E_r/kT) + \left(\frac{8}{\pi M}\right)^{1/2}(kT)^{-3/2}\int\sigma_{\mathrm{nr}}(E)E\exp(-E/kT)dE\,, \tag{1}$$

where the first term represents the contribution to the interaction rate from compound-nucleus resonances and the second term is the nonresonant contribution. Reference should be made to FCZ for numerical evaluation of the constants appearing above; $g_{\mathrm{Si}}{}^r = [2\,J_r(^{28}\mathrm{Si}) + 1]$ is the statistical-weight factor for the rth resonance in ^{28}Si, $g_p = 2$, and $g_{\mathrm{Al}}{}^i$ is the statistical-weight factor for the ith state of ^{27}Al. The nonresonant cross-section is denoted by $\sigma_{\mathrm{nr}}(E)$; Γ_γ is the *total* electromagnetic transition width to all final ^{28}Si states, for each resonance, as measured in the laboratory. A minor correction to equation (1), the enhancement of Γ_γ over the value measured in the laboratory by induced emission under stellar conditions, is insignificant at the temperatures expected for silicon burning and has been ignored.

Equation (1) does not include the effect of excited states in the $(^{27}\mathrm{Al} + p)$-system. If $\langle\sigma v\rangle^i$ represents the rate for ^{27}Al in the ith excited state to interact with protons, the total interaction rate for $^{27}\mathrm{Al} + p$ becomes

$$\langle\sigma v\rangle_{p\gamma} = \frac{\Sigma_i g_{\mathrm{Al}}{}^i \exp[-E_i(^{27}\mathrm{Al})/kT]\langle\sigma v\rangle_{p\gamma}{}^i}{G_{\mathrm{Al}}}\,, \tag{2}$$

where $E_i(^{27}\mathrm{Al})$ is the excitation energy for the ith state of ^{27}Al and G_{Al} is the partition function for the ^{27}Al nucleus, defined by

$$G_{\mathrm{Al}} = \Sigma_i g_{\mathrm{Al}}{}^i \exp[-E_i(^{27}\mathrm{Al})/kT]\,. \tag{3}$$

Data are available only for the calculation of $\langle\sigma v\rangle^0$ for the reactions considered here. At the temperatures of interest, radiative-capture rates from the excited states, $\langle\sigma v\rangle^{i>0}$, affect $\langle\sigma v\rangle_{p\gamma}$ only slightly since the excited states of ^{27}Al and ^{24}Mg are weakly populated (at $T_9 = 5$, 90 percent of the ^{27}Al and 83 percent of the ^{24}Mg nuclei are in their ground states).

Approximate values for $\langle\sigma v\rangle^i/\langle\sigma v\rangle^0$ were calculated from estimates of the average cross-sections for the excited state interactions (Lyons 1969*b*). "Black nucleus" transmission functions were used to obtain the nuclear-strength functions (see, for example, Vogt 1969), and the formulae for nuclear-level density given by Gilbert and Cameron (1965) were employed. Such calculations are only approximate at best, but since the calculated ratios of $\langle\sigma v\rangle^i/\langle\sigma v\rangle^0$ were small compared with unity (for $T_9 = 5$, the calculation yielded $\langle\sigma v\rangle_{p\gamma}{}^1/\langle\sigma v\rangle_{p\gamma}{}^0 \approx 0.27$, $\langle\sigma v\rangle_{p\gamma}{}^2/\langle\sigma v\rangle_{p\gamma}{}^0 \approx 0.12$, and $\langle\sigma v\rangle_{\alpha\gamma}{}^1/\langle\sigma v\rangle_{\alpha\gamma}{}^0 \approx 0.04$), a good approximation is $\langle\sigma v\rangle_{p\gamma} = g_{\mathrm{Al}}{}^0\langle\sigma v\rangle_{p\gamma}{}^0/G_{\mathrm{Al}}$ and $\langle\sigma v\rangle_{\alpha\gamma} = g_{\mathrm{Mg}}{}^0\langle\sigma v\rangle_{\alpha\gamma}{}^0/G_{\mathrm{Mg}}$.

Excited states in the $(^{27}\mathrm{Al} + p)$- and $(^{24}\mathrm{Mg} + \alpha)$-systems also contribute to the photodisintegration rate of ^{28}Si. In general,

$$\lambda_\gamma = \Sigma_i(\lambda_{\gamma p_i} + \lambda_{\gamma\alpha_i})\,. \tag{4}$$

If there is thermal equilibrium among the excited states, generalization of FCZ equations (10), (12), and (16) for the (γ,p) reaction yields

$$\lambda_{\gamma p}=\frac{G_{\mathrm{Al}}G_p}{G_{\mathrm{Si}}}\left(\frac{MkT}{2\pi\hbar^2}\right)^{3/2}\langle\sigma v\rangle_{p\gamma}\exp\left(-Q/kT\right). \tag{5}$$

Numerically, $G_p = 2$, $G_{\mathrm{Al}} \approx 6 + 2\ \exp\ (-9.782/T_9) + 4\ \exp\ (-11.756/T_9)$, and $G_{\mathrm{Si}} \approx 1 + 5\ \exp\ (-20.642/T_9)$.

The expression given for the (γ,p) reaction carries over directly to the (γ,α) reaction with obvious changes in statistical weights, reduced mass, and Q-value. Numerically, $G_{\mathrm{Mg}} \approx 1 + 5\ \exp\ (-15.883/T_9)$.

b) Data Used for the ^{27}Al$(p,\gamma)^{28}$Si *Rates*

Paper I presents the total resonance strengths and upper limits to nonresonant cross-sections for the proton-energy range 0.3–2.6 MeV. These data have been supplemented with data obtained by other investigators, outside this energy range. The tables of Endt and Van der Leun (1967) list three resonances below 0.3 MeV; these have been included in the rate calculations. Two groups have studied the reaction at higher energies: Gove, Litherland, and Batchelor (1961) have obtained cross-sections for the (p,γ_0) and (p,γ_1) reactions from $E_p = 3.8$–10.4 MeV, and Singh *et al.* (1965) have measured the (p,γ_0), (p,γ_1), and (p,γ_{2+3}) cross-sections from 4.0–12.5 MeV. In the region of overlap, the more detailed results of the latter paper were used.

In the energy range from 2.6 to 3.8 MeV, no (p_0,γ) data of sufficient detail are available, and the following approximation was used. The data of Paper I were averaged over 200-keV intervals, and the values thus obtained were compared with the data of Gove *et al.* and Singh *et al.* above 3.8 MeV. The energy-averaged total cross-section in the region 2.2–2.6 MeV ($\approx 100\ \mu$b) was approximately the same as that above 3.8 MeV. A value of $\sigma = 125 \pm 100\ \mu$b was therefore assumed for the region $E_p = 2.6$–3.8 MeV. A lower limit of $\sigma \approx 25\ \mu$b in this energy region was extracted from the (γ_0,p_0) work of Ullrich (1964). The uncertainty introduced into $\langle\sigma v\rangle^0$ by the cross-section assumed for the range $E_p = 2.6$–3.8 MeV is only 4.5 percent at $T_9 = 5$, and even smaller at lower temperatures. The upper limit to the nonresonant yield contributed only 3 percent to the rate at $T_9 = 5$, and less at lower temperatures.

c) Data Used for the ^{24}Mg$(\alpha,\gamma)^{28}$Si *Rates*

Paper II presents the total resonance strengths for α-particle energies below 2.8 MeV; nonresonant cross-sections could not be extracted because of the background produced by the ^{13}C$(\alpha,n)^{16}$O reaction. The data of Smulders and Endt (1962) were used in the range of α-particle energies 2.8–3.2 MeV. From $E_\alpha = 3.2$–4.5 MeV, only γ_0 and γ_1 data are available from the work of Weinman, Meyer-Schützmeister, and Lee (1964); no correction was applied to their data for $\gamma_{\geq 2}$ transitions. From $E_\alpha = 5.3$–14.5 MeV, the γ_0 and γ_1 transitions have been studied by Meyer-Schützmeister *et al.*, (1968); again, no correction for $\gamma_{\geq 2}$ transitions was included.

No (α_0,γ) data are available for $E_\alpha = 4.4$–5.2 MeV; however, (γ_0,α_0) data are available from bremsstrahlung studies (Ullrich 1964). These data were converted to (α_0,γ_0) cross-sections and normalized to the data of Meyer-Schützmeister *et al.* (1968). As a rough correction for $\gamma_{\geq 1}$ transitions, $\langle\sigma\rangle_{\alpha_0\gamma} = 2\langle\sigma\rangle_{\alpha_0\gamma_0}$ was used, for the range $E_\alpha = 4.4$–5.2 MeV. The resulting uncertainty in the total (α_0,γ) cross-sections for $E_\alpha \geq 3.2$ MeV is considerable. However, for $T_9 = 2$, 4, and 5, the resonances above 3.2 MeV contribute only 0.15, 7, and 14 percent, respectively, of the total interaction rate. In addition, since the (γ,α) channel constitutes only about 10 percent of the total rate of ^{28}Si photodisintegration at $T_9 = 5$, the uncertainty does not seriously impair the precision of the derived astrophysical photodisintegration rates, at least up to $T_9 = 5$.

d) Empirical Fits to Interaction Rates

Numerical calculations of the interaction rate multiplied by Avogadro's number, $N_A\langle\sigma v\rangle^0$, were made with the data discussed in §§ II*b* and II*c*, and these are presented in Table 1 for selected temperatures. For most astrophysical purposes, however, semiempirical fits to the calculated rates are more easily applied. Various semiempirical expressions have been tried by FCZ, who found that the form $\langle\sigma v\rangle = A \exp(-E_t/kT)$ yields satisfactory fits for $T_9 \geq 1$. This implies $\sigma(E) \propto (E - E_t)^{1/2}/E$ for $E > E_t$. The quantity E_t is the energy above which the barrier-penetration effects on Γ_p become unimportant in determining the resonance γ-ray yield, i.e., $\Gamma_p \approx \Gamma$ for $E > E_t$. It is clear that this functional form for $\langle\sigma v\rangle^0$ will not be adequate at lower temperatures, where the small penetration factor makes $\Gamma_p \ll \Gamma$.

A least-squares fit of the interaction rates of Table 1 for $T_9 = 1.0$–5.0 to the expression of FCZ gave, for the $^{27}Al(p,\gamma)^{28}Si$ interaction rate,

$$N_A\langle\sigma v\rangle^0 = 8.19 \times 10^4 \exp(-6.342/T_9) . \tag{6}$$

This expression fits the data of Table 1 with a maximum deviation of 14 percent over the range $T_9 = 1.0$–5.0, but at $T_9 = 0.5$, this fit is about a factor of 8 below the numerical results of Table 1.

TABLE 1

STELLAR INTERACTION RATES*

Temperature (10^9 ° K)	$N_A\langle\sigma v\rangle^0$ $^{27}Al(p,\gamma)^{28}Si$	$N_A\langle\sigma v\rangle^0$ $^{24}Mg(\alpha,\gamma)^{28}Si$	Temperature (10^9 ° K)	$N_A\langle\sigma v\rangle^0$ $^{27}Al(p,\gamma)^{28}Si$	$N_A\langle\sigma v\rangle^0$ $^{24}Mg(\alpha,\gamma)^{28}Si$
0.1	1.43×10^{-8}	3.27×10^{-56}	1.0	$1.66\times10^{+2}$	6.06×10^{-4}
0.2	8.47×10^{-4}	2.50×10^{-27}	1.5	$1.07\times10^{+3}$	6.64×10^{-2}
0.3	4.25×10^{-2}	9.12×10^{-18}	2.0	$3.01\times10^{+3}$	7.48×10^{-1}
0.4	4.27×10^{-1}	6.25×10^{-13}	2.5	$5.91\times10^{+3}$	3.45×10^{-0}
0.5	2.19×10^{-0}	5.70×10^{-10}	3.0	$9.46\times10^{+3}$	$1.02\times10^{+1}$
0.6	7.67×10^{-0}	5.68×10^{-8}	3.5	$1.34\times10^{+4}$	$2.35\times10^{+1}$
0.7	$2.09\times10^{+1}$	1.55×10^{-6}	4.0	$1.75\times10^{+4}$	$4.48\times10^{+1}$
0.8	$4.75\times10^{+1}$	1.86×10^{-5}	4.5	$2.17\times10^{+4}$	$7.58\times10^{+1}$
0.9	$9.38\times10^{+1}$	1.29×10^{-4}	5.0	$2.58\times10^{+4}$	$1.17\times10^{+2}$

* Due to uncertainties in the experimental data, standard deviations of 15–20 and 25–30 percent should be assigned to the $^{27}Al(p,\gamma)^{28}Si$ and $^{24}Mg(\alpha,\gamma)^{28}Si$ rates, respectively.

For the $^{24}Mg(\alpha,\gamma)^{28}Si$ reaction, the best fit for the range $T_9 = 1.0$–5.0 was given by

$$N_A\langle\sigma v\rangle^0 = 1.85 \times 10^3 \exp(-15.163/T_9) , \tag{7}$$

which fits the rates given in Table 1 with a maximum deviation of 26 percent over this temperature range. At $T_9 = 0.5$, this fit predicts a rate that is about a factor of 4 below that of Table 1.

In order to extend the useful temperature range, four-parameter fits of the form $A \exp(-B/T) + C \exp(-D/T)$ were made. For the $^{27}Al(p,\gamma)^{28}Si$ case, the form

$$N_A\langle\sigma v\rangle^0 = 9.61 \times 10^4 \exp(-7.305/T_9) + 4.61 \times 10^3 \exp(-3.857/T_9) \tag{8}$$

fits the rates given in Table 1 with a maximum deviation of 5 percent over the entire range $T_9 = 0.5$–5.0. In addition, the rates given in Table 1 have a standard deviation of about 15–20 percent which arises from the uncertainties in the experimental data.

For the $^{24}Mg(\alpha,\gamma)^{28}Si$ case,

$$N_A\langle\sigma v\rangle^0 = 3.39 \times 10^3 \exp(-19.575/T_9) + 6.21 \times 10^2 \exp(-13.863/T_9) \tag{9}$$

fits the rates given in Table 1 with a maximum deviation of 9 percent for $T_9 = 0.5$–5.0. For ^{24}Mg(α,γ)^{28}Si, the rates given in Table 1 have a standard deviation of about 25–30 percent.

The photodisintegration rate $\lambda_\gamma(^{28}\mathrm{Si})$ may not be the most important rate-determining parameter for silicon burning; in the analysis of Bodansky *et al.* (1968), $\lambda_\gamma(^{24}\mathrm{Mg})$ is the most critical parameter. In their study, the $^{28}\mathrm{Si} + \gamma \rightleftarrows \alpha + {}^{24}\mathrm{Mg}$ reactions come into approximate equilibrium; thus, the rate at which α-particles and nucleons are made available as subsequent building blocks is determined by the leakage out of this approximate equilibrium, and is therefore proportional to $\lambda_\gamma(^{24}\mathrm{Mg})$. The experimental apparatus and techniques developed for the ^{28}Si work described in Papers I and II are currently being used to study the ^{24}Mg photodisintegration rate.

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